Polarization Study

1 Introduction

Polarization effects can be confusing — even counter intuitive if you have the wrong intuition. To add to the confusion, the convention to define "left" and "right" circular polarization is . . . at the whim of your instructor. An atomic physicist will describe polarization of light with respect to a *fixed* "z-axis" of reference, independent of the **k**-vector of the light. This definition becomes somewhat of a challenge when that axis is orthogonal or opposite to the direction of propagation of light. In Optics, the polarization is defined with respect to the **k**-vector. It is obviously important to know which definition to use when discussing interaction of radiation and matter.

Polarization can be "manipulated" in the laboratory by reflection off a metallic or dielectric interfaces, and transmission through birefringent materials[1] and through media with circular dichroism[2]. Polarization is manipulated on paper with Jones or Mueller matrices. See Appendix **??** for a short summary of Jones matrices. A basic optics text such as Hecht's [1] is sufficient to get you started. Other good sources include Refs. [2] and [3].

1.1 Maintaining linear polarization

Numerous experiments in optics are based on polarization selection. For instance, you may want to combine a strong pump pulse with a weak probe. The technique is to make the two beams orthogonally polarized, and combine them via a polarizing beam splitter. After they have propagated through a sample under analysis, another polarizing beam splitter will be used to separate them. Such a trivial "Gedanken experiment" can be the experimentalist worst nightmare, if the pump has a very high intensity as compared to the probe. The problems that one might encounter are the following:

1. Depolarization on reflection: the polarization is maintained if the beam is polarized *exactly* in the plane of incidence (p polarization), or *exactly* orthogonal to the plane of polarization. There is however a different phase shift upon reflection for s and p incidence, which has a consequence that linear polarization will become elliptical if the initial polarization is not aligned. This effect is less pronounced for metallic reflector, which is the reason metallic corner cubes are used in the wavemeter experiment.

2. Geometrical effect: In a dielectric (hence in air) the polarization vector is orthogonal to the k vector. In any polarization sensitive experiment, the beam should be maintained either in a plane containing the polarization vector, or a plane orthogonal to the polarization vector [see question (a) at the end of this chapter]

3. Elements to manipulate the polarization (waveplates, polarizers) are generally wavelength dependent, and may not operate over the full bandwidth of the laser pulse

4. The slightest stress on a mirror, window can significantly alter the polarization of the transmitted beam.

[1]Media with different indices along two orthogonal axes.

[2]Media with different indices for right- and left-circularly polarized light.

1.2 Maintaining circular polarization

If one considers a circular polarized beam as resulting from the combination of two linearly polarized beams of orthogonal polarization, dephased by $\pi/2$, one can see that all the problems mentioned above still apply. Conditions are however more stringent to maintain circular polarization: because the phase shift upon reflection is different in s and p polarization, circular polarization is not conserved under other than normal incidence. This can be understood without any other consideration than conservation of angular momentum. With positive circularly polarized light, $\sigma+$, the electric field vector rotates at an angular velocity $+\omega$ around the $\mathbf{k}$ vector. One would expect the angular momentum of the radiation to be conserved upon reflection. If the deflection is small (i.e at grazing incidence), the electric field is expected to continue spinning in the same direction around the slightly deflected $\mathbf{k}$ vector, and the polarization is conserved. For near normal reflection however, because the $\mathbf{k}$ vector has reversed sign, what was a positive direction of rotation $+\omega$ becomes $-\omega$ for the reflected axis. The polarization state is reversed. Which of the above two scenarios is the correct one? You will find that both limits are correct. The transition from $\sigma+$ to $\sigma-$ occurs at an angle of incidence which depends on the details of the reflection.

2 Theory

2.1 Polarization and reflection

We begin by going back to basic refraction-reflection theory as discussed for example in Chapter 13 of Born and Wolf [4]. One recalls first of all that Snell's law applies. It relates the angle of refraction θ_2 to the angle of incidence θ_1 of a wave entering a medium with refractive index n_2 from a medium with refractive index n_1:

$$n_2 \sin(\theta_2) = n_1 \sin(\theta_1). \tag{1}$$

Note that in general both refractive indices can be complex numbers and care must be taken when interpreting the meaning of the (complex) angles.

Secondly, Fresnel's equations can be used to obtain the reflection coefficients. For an incident electric field of amplitudes $A_{\|}$ and $A_{\perp}$ ($\perp = s = \sigma$ polarization and $\| = p = \pi$ polarization), the amplitude of the reflected fields $A'_{\|}$ and $A'_{\perp}$ are given by:

$$A'_{\|,\perp} = r_{\|,\perp} A_{\|,\perp} \tag{2}$$

with the amplitude reflection coefficients

$$\begin{aligned} r_{\|} &= \frac{n_2 \cos(\theta_1) - n_1 \cos(\theta_2)}{n_2 \cos(\theta_1) + n_1 \cos(\theta_2)} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}, & (3) \\ r_{\perp} &= \frac{n_1 \cos(\theta_1) - n_2 \cos(\theta_2)}{n_1 \cos(\theta_1) + n_2 \cos(\theta_2)} = -\frac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)}. & (4) \end{aligned}$$

Note that the amplitude reflection coefficients are in general complex:

$$\begin{aligned} r_{\|} &= \rho_{\|} e^{i\phi_{\|}}, & (5) \\ r_{\perp} &= \rho_{\perp} e^{i\phi_{\perp}}. & (6) \end{aligned}$$

Suppose the incident beam is linearly polarized with σ and π polarization components of amplitude $A_\parallel$ and $A_\perp$. An azimuthal polarization angle α_i can be defined by

$$\tan(\alpha_i) \;=\; \frac{A_\perp}{A_\parallel}. \tag{7}$$

The corresponding reflected beam will have an azimuthal angle α_r obeying the relations

$$\tan(\alpha_r) \;=\; \frac{A'_\perp}{A'_\parallel} \;=\; \frac{r_\perp}{r_\parallel}\frac{A_\perp}{A_\parallel} \;=\; -\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_1+\theta_2)}\tan(\alpha_i) \;=\; Pe^{-i\Delta}\tan(\alpha_i) \tag{8}$$

with

$$\mathrm{P} = \frac{\rho_\perp}{\rho_\parallel} \tag{9}$$

and

$$\Delta = \phi_\parallel - \phi_\perp. \tag{10}$$

What does all of this mean? Looking at Eq. (8), we see that for phase delay differences Δ that are not an integer multiple of π, $\tan(\alpha_r)$ is imaginary. This means that the reflection induces a phase difference between the two orthogonal polarization components $A'_\parallel$ and $A'_\perp$. As a result, linearly polarized light becomes elliptically polarized. There are two noteworthy cases where $\tan(\alpha_r)$ is real and the light remains linearly polarized upon reflection:

1. Normal Incidence: $\theta_1 = 0$, which results in $\mathrm{P} = 1$ and $\Delta = -\pi$,
2. Grazing Incidence: $\theta_1 = \pi/2$, which results in $\mathrm{P} = 1$ and $\Delta = 0$.

These two cases are obvious if we consider the linearly polarized light to be made of two circular polarization components of opposite rotation. At normal incidence, the reflected components are interchanged (right becomes left, left becomes right). At grazing incidence, the two circular components are conserved. At any other incidence the reflection of a circular component is elliptic, and can be considered "in transition" from one sense of rotation to the other.

From Eq. (8) it is also obvious that light remains linearly polarized if both θ_1 and θ_2 are real. For the general case, one expects $-\pi \le \Delta \le 0$. A case of special importance is when $\Delta = \pm\frac{\pi}{2}$, for which one defines $\theta_1 = \bar{\theta}_1$ as the principal angle of incidence. Typically, the reflected light will be elliptically polarized in this case (with the oscillation directions for the electric field being parallel and perpendicular to the plane of incidence). However, if $\mathrm{P}\cdot\tan(\alpha_i) = 1$, then $\tan(\alpha_r) = \pm i$, making the reflected light circularly polarized.

Let us suppose we perform an experiment where linearly polarized light with $\alpha_i = \pi/2$, $A_\perp = A_\parallel = A$ propagates in air ($n_1 = 1$) and is reflected by a material of unknown (complex) index n_2. In general, after reflection the light is elliptically polarized with a polarization ellipse as shown in Fig. 1. From a measurement of $\rho_\perp$, $\rho_\parallel$ and β on can determine P and Δ. To show this we need to analyze the polarization state of the reflected light in more detail. The electric field components after reflection can be written as

$$E'_\perp \;=\; A'_\perp\cos\omega t \tag{11}$$
$$E'_\parallel \;=\; A'_\parallel\cos(\omega t+\Delta). \tag{12}$$

It is easy to show that these components satisfy the equation

$$\left(\frac{E'_\perp}{A'_\perp}\right)^2 + \left(\frac{E'_\parallel}{A'_\parallel}\right)^2 - 2\frac{E'_\perp E'_\parallel}{A'_\perp A'_\parallel}\cos\Delta = \sin^2\Delta \tag{13}$$

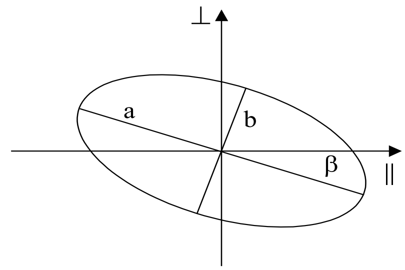

Figure 1: Polarization ellipse.

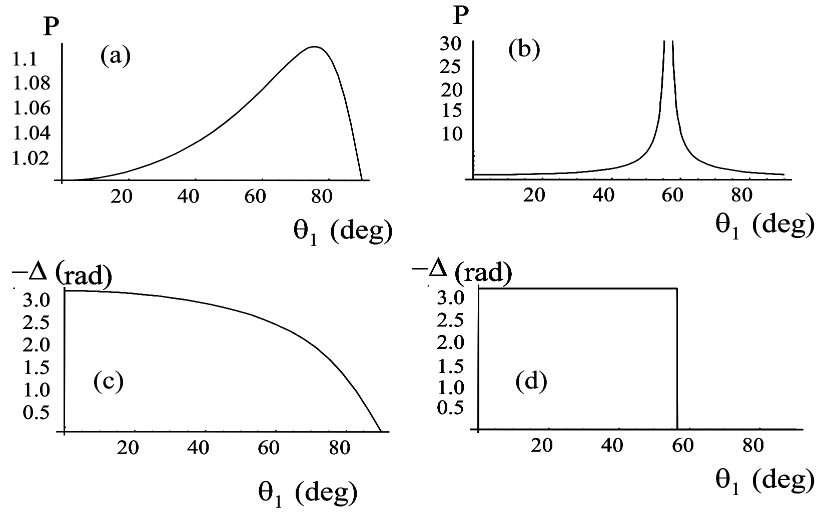

Figure 2: a) P vs θ_1 for a typical metal; b) P vs θ_1 for a typical glass; c) $-\Delta$ vs θ_1 for a typical metal; d) $-\Delta$ vs θ_1 for a typical glass.

This is the equation of an ellipse rotated by an angle β with respect to the coordinate frame $(\perp, \|)$. The angle β is related to the parameters of Eq. (13) by

$$\tan(2\beta) = \frac{2A'_\perp A'_\|}{A'^2_\| - A'^2_\perp} \cos\Delta \tag{14}$$

With P and Δ known one can determine n_2. This is the basic principle of a technique called ellipsometry. As an exercise you should derive Eqs. (13) and (14).

Based on Eqs. (1,3,4), one can determine theoretical curves for P and Δ as shown in Fig. 2. The left figures are typical of reflection from a pure metal whereas the right ones are from a typical dielectric (glass). Note that the θ_1 value corresponding to the maximum of the "P" plot for metals matches up with the $\bar{\theta}_1$. In the case of glass, the "P" function "blows up" rather than peaks at $\theta_1 = \theta_B =$ Brewster's angle.

Note that the plots above apply to *reflection at an air-material interface.* Do not confuse with the situation of the Fresnel Rhomb, where the reflection is Total Internal Reflection (TIR) and a *glass-air interface.*

3 Experiments

3.1 Aligning the polarizers

In your experiments you will use several polarizers whose pass directions are unknown (your laser is unpolarized). Therefore you need to calibrate the reading on the rotation stages holding the polarizers with respect to the laboratory frame ($\|, \perp$). One way to accomplish this is to take advantage of the reflection of light off an uncoated glass slide at Brewster's angle. At Brewster's angle, $A'_{\|} = 0$. Since the refractive index of the glass slide is not known exactly you need to find a combination of the polarizer angle and angle of incidence for which $A'_{\|} \approx 0$. Another option is to take advantage of the bright polarized sky of New Mexico.

3.2 Waveplates

Waveplates are the most commonly used devices for adjusting polarizations but they are often misunderstood. To gain a better intuition for these tools, one can do some simple experiments.

Starting with linearly polarized light, place a half-wave plate (HWP) and a polarizer (set so that its transmission axis is "crossed" with respect to the initial polarization of the light) in the beam. Adjust the HWP so that it has no effect. Take the rotation stage readings for both the polarizer and the HWP. Adjust the angle of the HWP and then adjust the polarizer until no light is passed through the pair. This should verify to you that the HWP rotates the polarization angle of the linear polarized light. (Use Jones matrices to see how the HWP act on elliptically polarized light of a given helicity) Now vary the two orientations so that you can plot the change in the angle of the linear polarization as a function of the HWP angle. How does this agree with theory?

Do a similar test on a quarter-wave plate (QWP). Start with the laser polarization in the "half s, half p" ($\alpha_i = 45°$) orientation. Place a QWP in the beam and orient it to get circularly polarized output. Once this is done, place a polarizer and detector (or power meter) after the QWP. For this fixed QWP setting, plot the power throughput as a function of the polarizer angle from 0° to 180°. Do a similar plot for the QWP being rotated by 22.5° and 45°. Explain your findings.

A more accurate adjustment of the QWP uses the property that two passages through a QWP should correspond to a HWP, rotating the polarization 90°. In an arrangement laser – polarizer – QWP – mirror, the reflected beam should be totally extinguished by the polarizer. If a green laser is available, repeat the measurement with that laser. Can you get total extinction? How? Why?

3.3 The Fresnel Rhomb

In many circumstances, we need to control the polarization of beams accurately (say for example to obtain optimal contrast in an interferometer). There are a number of optical elements that can be used for this purpose. Their common feature is to introduce a phase change between two orthogonally polarized light beams. This can be done, for example, by utilizing the optical birefringence in crystals (which leads to the waveplates discussed in the previous section) *or* using total internal reflection (TIR) at the *interface between glass and air*. Do not confuse TIR with "critical angle". TIR occurs at any angle above the critical angle. The reflection amplitude is then always unity (= 100%), but the phase shift is a steep function of the angle of incidence (above the critical angle). Review you "favorite" optics book for expressions of the phase shift

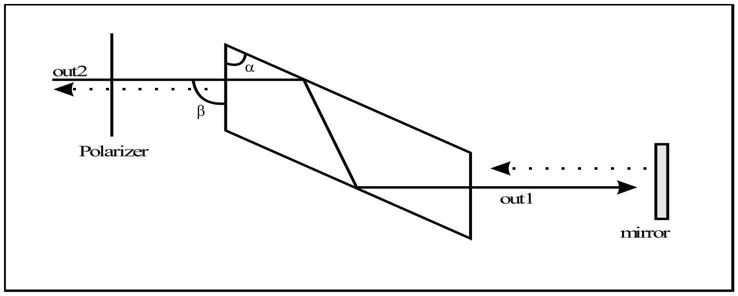

Figure 3: The Fresnel Rhomb

upon TIR. The latter method is exploited in the so-called Fresnel rhomb (see Fig. 3, which has the advantage of a broad bandwidth. The disadvantage is that the beam is transmitted to a significant amount of glass, resulting in pulse distortion by dispersion, and a possibility of laser damage by self-focusing.

The incident light is linearly polarized with the axis tilted 45° with respect to the edges of the rhomb's input face (i.e. half $\perp$, half $||$). The light undergoes two total reflections before it leaves the rhomb each giving a $\pi/4$ phase shift. *Again, total reflection does not mean that you have to be at the critical angle.*

1. Determine the refractive index n which the glass has to have for the output light to be circularly polarized. Assume a symmetrical beam path as shown in Fig. 3 (with $\beta = 90°$) and $\alpha = 45°$. You should be able to perform this calculation analytically. *again, you are* **above** *critical angle, and you should use the appropriate formulae for this situation.*

2. In the experiment the rhomb angle $\alpha \neq 45°$ and the refractive index of the glass at the laser wavelength are unknown. As a consequence, the rhomb has to be rotated slightly to produce circularly polarized light (i.e. $\beta \neq 90°$ or $\beta = 90° + \Delta\beta$). You can experimentally determine the index, by adjusting the rhomb prism orientation to make it a perfect quarter wave element. You have also several other techniques to determine the index of refraction.

   a Brewster angle

   b Minimum deviation

   c Make one face normal to the He-Ne beam, and measure the deviation angle.

   d Polarization, as outlined below.

3. Measure the incident angle β for which the light at (out 1) is circularly polarized and the rhomb angle α. Make sure that the polarization of the incident laser is linear and the polarization axis is tilted 45° with respect to the corresponding edges of the rhomb. To decide when the light is circularly polarized, it is convenient to retro-reflect the light back so that it passes through the rhomb device again. In the perfectly aligned case the rhomb then acts as a $\lambda/2$ plate, rotating the polarization of the input light by 90°. Thus we expect no light at "out 2" since the polarizer P blocks polarizations orthogonal to that of the input.

4. From the measurement of β and α determine the actual refractive index n of the rhomb. Plot the function $n(\Delta\beta)$ in the interval $(\Delta\beta_0 - 10°, \Delta\beta_0 + 10°)$ where $\Delta\beta_0$ is the angle

you measured. Do not forget to consider refraction at the rhomb's input face if $\Delta\beta \neq 0$! These calculations cannot be done analytically. *Again, do not confuse glass-air interface with air-glass interface.*

5. As a verification of the index n you determine, you should try at least one other quick method. One possibility is to simply measure the Brewster angle and to obtain n from it. This method typically is not very accurate. Why?

   Another method is to use the tip of the rhomb as a prism and to apply the principle of minimum deviation (see, for example [5]). For a beam incident into the prism-rhomb tip, varying the angle of incidence into the "prism" will give a fairly distinct minimum deviation output angle. This well-defined angle can be used to calculate the refractive index n directly.

3.4 Depolarization upon reflection

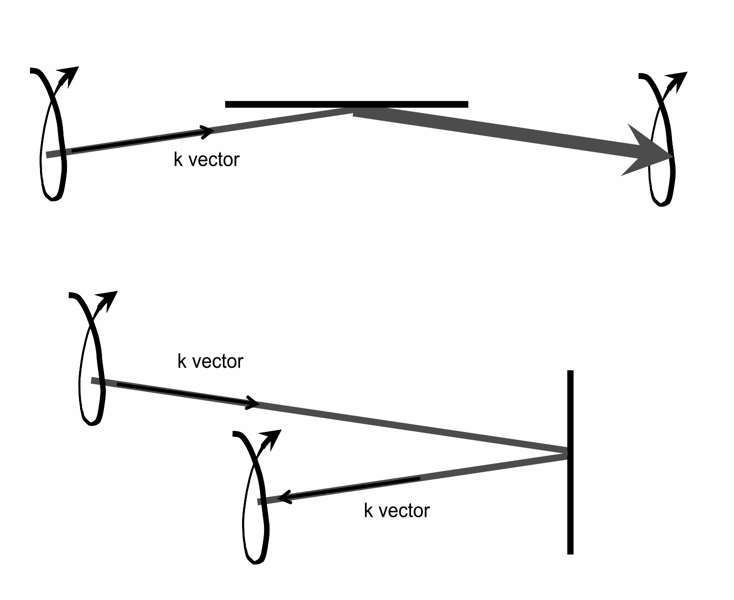

Figure 4: Circular polarization is conserved at grazing incidence (top), and reversed near normal incidence (bottom).

This experiment is intended to illustrate polarization effects upon reflection. The Fresnel Rhomb was one example of how the polarization can be affected by internal reflection off of a dielectric material/air interface. However, does reflection from a mirror introduce any polarization changes?

A simple experiment illustrated in Fig. 4 indicate that there should be a dependence. The top figure shows a right circularly polarized beam propagating from left to right, and gets reflected at grazing incidence. Grazing incidence or going straight is nearly the same thing. If the right spinning field of photon has any inertia, one would expect the right circular polarization to be conserved. If however, as shown on the bottom of the figure, the reflection is near normal, one would use the same argument to claim that the field continues to spin in the same direction in the laboratory frame. However, the k vector of the light has reversed, and therefore the polarization of the reflected beam is now left circular (in optics, the reference axis is always the k vector). Therefore, as the angle of incidence on the mirror is changed from normal (zero) to grazing (90 ^{o}), there should be at some angle a change of polarization of the reflected beam from left circular to right circular. This can be measured experimentally, but... it is a difficult experiment. Circular polarization can be decomposed into two orthogonal linear polarizations with a relative phase of $\pi/2$. Measuring the change of phase upon reflection of linear polarization, as a function of angle of incidence, provides the same information as the experiment on circular polarization.

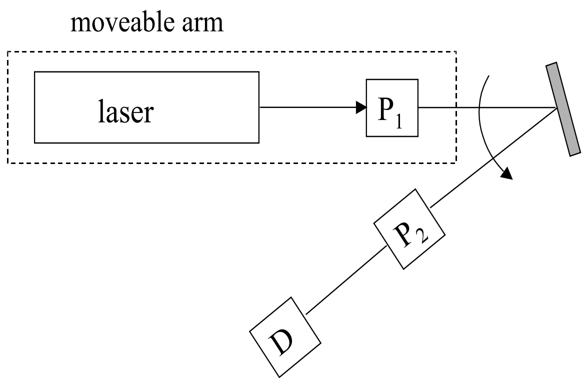

Figure 5: Experimental setup to measure the depolarization upon reflection. The light after polarizer P_1 should be linearly polarized with $\alpha_i = 45°$.

To answer this question you will determine P and Δ as a function of the angle of incidence for two surfaces (choose one metal surface and a dielectric multi-layer mirror). First produce a beam that is linearly polarized with an azimuthal polarization angle $\alpha_i = 45°$. Plot the intensity transmitted through a linear polarizer as a function of its orientation. Repeat this experiment with the beam reflected off a surface for several angles of incidence. To this end, mount the laser on the moveable arm and make sure that the rotational axis is in the plane of the reflecting surface. This will prevent a lateral displacement of the beam during rotation.

From the polarizer angle (polarizer P_2) at which maximum transmission occurs, you can determine the angle of the polarization ellipse β, cf. Fig. 1. Measure the reflected light amplitudes $A'_\perp$ and $A'_\parallel$ by rotating your polarizer parallel to $\perp$ and $\parallel$, respectively. Use Eq. (14) and Eq. (9) to determine P and Δ. Plot both quantities as a function of the angle of incidence. Interpret your results and compare them to the theoretical expectations if possible.

Optional: As an optional exercise, derive equations that allow you to determine P and Δ from the measured β and the maximum and minimum field amplitudes that you obtain while rotating the polarizer. Obviously the latter are the amplitude components parallel to the axes of the ellipse. For one material (metal), determine the complex refractive index, n_2.

4 Questions

(a) Find an optical arrangement to rotate by 90 degrees the linear polarization of a beam, using only mirrors.

(b) Verify how waveplates affect the light's polarization, both on paper and experimentally.

(c) Check the action of the Fresnel Rhomb on polarized light.

(d) Measure the refractive index n of the Fresnel Rhomb in a number of different ways.

(e) Study how reflection affects the different incident polarizations.

(f) Design a quarter wave plate that will transform a linearly polarized beam into a circularly polarized one, with less that 10% ellipticity over the bandwidth of a 50 fs pulse at 266 nm. Can you achieve the same goal with a fused silica Rhomb prism?

(g) Can you use the quarter wave plate of the previous problem at 300 nm?

5 Related exam problem

Consider a quartz plate of 1.054 mm thickness. The optical axis of the crystal is in the plane of the plate.

Radiation at $\lambda_0 = 800$ nm is sent through that plate, with its (linear) polarization oriented at 45° with respect to the optics axis.

The k vector magnitudes for quartz and their derivatives at the frequency corresponding to 800 nm are:

	k μm^{-1}	$\frac{dk}{d\Omega}$ s μm^{-1}	$\frac{d^2k}{d\Omega^2}$ $\text{s}^2\mu\text{m}^{-1}$
extraordinary	12.15	$5.21 \cdot 10^{-15}$	$4.21 \cdot 10^{-32}$
ordinary	12.08	$5.18 \cdot 10^{-15}$	$4.13 \cdot 10^{-32}$

A. What is the state of polarization of radiation at 800 nm after transmission?

Consider next that the incident light has a spectrum that extends over a certain bandwidth. In this bandwidth, consider two frequencies at $\Omega_{1,2} = \Omega_0 \pm \Delta\Omega$ where $\Delta\Omega = 0.0469 \cdot 10^{15}$ s^{-1}, and Ω_0 is the angular frequency corresponding to 800 nm.

B. What is the state of polarization of the transmitted radiation at each of the two frequencies Ω_1 and Ω_2?

C. What are the transit times, at the group velocity, for wave packets centered at each of the two frequencies Ω_1 and Ω_2, for an input radiation polarized along the optic axis (e polarization)? Find the difference between these two transit times for both "e" and "o" polarizations.

Let us consider next that this spectrum is that of a Gaussian pulse (bandwidth limited), centered at 800 nm, with the electric field amplitude given by

$$\mathcal{E}(t) = \mathcal{E}_0 e^{-(\frac{t}{\tau_G})^2}.$$

D. Knowing that $\Omega_2 - \Omega_1 = 2\Delta\Omega$ represents the Full Width at Half Maximum of the spectral intensity of the incident pulse, find τ_G.

E. In view of the above results, describe the transmitted pulse with a sketch and a few sentences. From (C), explain a change in pulse temporal behavior (output pulse width, as compared to the input pulse width). From (A) and (B), describe the evolution of the output pulse polarization versus time.

References

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